Deriving the Coordinates for the Vertex of a Parabola Using Completing the Squares Method

Start with the general form for a parabola, $y = ax^2 + bx + c$, and manipulate into the form $y = a(x - h)^2 + k$, where (h,k) are the coordinates of the vertex:

$y = ax^2 + bx + c$

$\Rightarrow y - c = ax^2 + bx$

$\Rightarrow y - c = a(x^2 + \frac{b}{a}x)$

Since $\frac{b}{2a} \rightarrow \frac{b^2}{4a^2}$

$y - c = a(x^2 + \frac{b}{a}x) \Rightarrow y - c + a(\frac{b^2}{4a^2}) = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2})$

$\Rightarrow y - c + \frac{b^2}{4a} = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2})$

$\Rightarrow y - \frac{4ac}{4a} + \frac{b^2}{4a} = a(x + \frac{b}{2a})^2$

$\Rightarrow y = a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + \frac{4ac}{4a}$

$\Rightarrow y = a(x - (-\frac{b}{2a}))^2 + (\frac{4ac - b^2}{4a})$

$\Rightarrow (h, k) = (-\frac{b}{2a}, \frac{4ac - b^2}{4a})$